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# Investment appraisal using quantitative risk analysis

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**Abstract**

Investment appraisal concerned with investments in fire safety systems is discussed. Particular attention is directed at evaluating, in terms of the Bayesian decision theory, the risk reduction that investment in a fire safety system involves. It is shown how the monetary value of the change from a building design without any specific fire protection system to one including such a system can be estimated by use of quantitative risk analysis, the results of which are expressed in terms of a Risk-adjusted net present value. This represents the intrinsic monetary value of investing in the fire safety system. The method suggested is exemplified by a case study performed in an Avesta Sheffield factory. © 2002 Elsevier Science B.V. All rights reserved.

*Keywords:* Decision analysis; Risk analysis; Investment appraisal; Fire protection; Bayesian updating

## 1. Introduction

Making a decision of whether to install a particular fire protection measure in a factory can be difficult, particularly if the measure is not required for meeting the demands of the building code in question. In such a situation, a method is needed for comparing the benefits the fire protection measure would provide with the costs of investing in it. Decision-making problems of this type are traditionally solved using some capital investment method, e.g. net present value or rate of return, in order to calculate the profitability of the investment, and it would be beneficial if a similar method could be used in the present context.

How should such a traditional investment appraisal method be employed in the present context in a way allowing the reduction in risk that the investment implies to be taken into account? One way is to evaluate the risk reduction in terms of its *intrinsic* monetary value, treating it as "income" from the investment in question. Estimating the intrinsic monetary value of the risk reduction that a specific fire safety investment provides can be based on the use of decision theory. This involves investigating the decision maker's preferences towards risk, identifying fire scenarios that are representative for the building, and employing some

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form of quantitative risk analysis (QRA) by which the probabilities and the consequences of the different fire scenarios are estimated.

The present paper proceeds with a short presentation of decision theory in the context of decisions on fire protection measures and a discussion of how the uncertainties concerning the probabilities involved can be handled in a decision analysis. An account is provided of how one can model the frequency of fire as well as the different fire scenarios that can occur in a given building so as to be able to estimate the intrinsic monetary value of the risk reduction achieved by investing in a specific fire safety measure. How the uncertainty here can be reduced by the use of fire statistics is also taken up. The paper concludes with the presentation of a practical application of the suggested method in a case study involving investment in a sprinkler system in the cold-rolling mill of the Avesta Sheffield plant in Nyby, Sweden.

## 2. Decision analysis

In this section a brief account of decision analysis in the present context of fire safety is provided (for a more detailed description, see Johansson [1]). The concept of *certainty equivalent* (CE) will be considered in some detail because of its importance to the model for the investment appraisal of fire safety measures suggested here.

Modern decision theory has its roots in particular in the work performed by Ramsey [2], Von Neumann and Morgenstern [3] and Savage [4], who have developed axiomatic systems for comparing preferences for different acts with uncertain outcomes. The basic approach taken in constructing such axiomatic systems has been to formulate various rules (axioms) that seem intuitively reasonable for comparing preferences between different acts with uncertain outcomes. From these axioms, a number of important principles can then be derived, such as the principle of maximising expected utility (MEU). The MEU principle implies that a person who is willing to follow these axioms in his/her decision-making will evaluate decision alternatives in terms of their expected utility and choose the alternative for which the expected utility is highest. Of the authors just referred to, Savage is the one often regarded as the principal founder of modern decision theory [5], or of what is also termed Bayesian decision theory. A review of various theories of this type has been provided by Fishburn [5].

Decision analysis, as described in a general way in [6], for example, involves the derivation of an utility function defined in terms of one or more attributes (such as monetary consequences, for example) that the decision maker wishes to take account of. The utility values obtained can be seen as measures of the decision makers' preferences, a consequence with a higher utility value being preferred to one with a lower value. Techniques for eliciting utility functions are summarised in [7].

When the decision maker's utility function has been determined one can calculate the expected utility of the different decision alternatives on the basis of the probabilities of the different consequences and their respective utility values. In the present context the word "disutility" might be considered more appropriate, since in most cases it is the utility of losses one is interested in. Nevertheless, the term utility will be used throughout. One should bear in mind, however, that it is usually a negative utility value that is meant when the expected utility of a fire is referred to.

It is important to note that the consequences of a fire are of a multi-attribute character. A serious fire can involve loss of sales, loss of market shares, getting a negative reputation, etc. Losses of this sort that the decision maker is not compensated will be termed *uncompensated losses*. It can be useful to express these in terms of their *intrinsic* negative monetary value ([1,8]). This allows measures of relative preference for the different possible sets of consequences to be obtained, and it gives the decision maker an effective means of communicating how good or bad he/she judges a particular outcome of a fire to be, since the monetary scale is one that people are accustomed to. The technique used for estimating the intrinsic negative monetary value of a specific loss can vary. It has been suggested that the technique adopted involves analysis at different levels that may differ considerably in the effort they require ([1,8]). In the approach advocated in the present paper, no general evaluation of the different attributes is made, evaluations being performed instead on a scenario basis, the decision maker expressing his/her preferences within the framework of each fire scenario.

In the present context, calculating the monetary value for the decision maker of the reduction in risk that a particular fire safety investment involves is of interest. In carrying out a quantitative fire-risk analysis for a building, one estimates the probability of each of the possible consequences both before and after the investment under consideration has been made, and expresses the consequences as utility values. This allows the expected utility, given that a fire has occurred in the building, to be calculated. This value, in turn, can be translated into a CE, which in the present case is the monetary value the decision maker is prepared to pay in order to escape the effects of an occurrence of fire in the building. A formal definition of CE is provided in Eq. (1), in which $u(\mathrm{CE})$ is the utility value corresponding to the monetary amount CE, $u(c_i)$ the utility value corresponding to the consequence $c_i$, and $n$ the number of possible consequences. A general definition of CE is "... the amount of money that is equivalent in your mind to a given situation that involves uncertainty" [6]. Assume that CE has been calculated both for the alternative in which the building is equipped with the fire safety measure under consideration and for the building in its current state. If, in addition to this, one has an estimate of the annual frequency of fire (expected number of fires per year), one can also estimate, for any given time period, the intrinsic monetary value of the reduction of risk that the investment involves.

$$u(\mathrm{CE}) = \sum_{i=1}^{n} u(c_i) \tag{1}$$

To illustrate how this can be done, assume that a decision maker has two alternatives for the fire protection to be found in a particular factory, the first being to keep the factory in its present state and the second to invest in a certain type of fire safety measure. Both alternatives can be regarded as "lotteries". The difference between this situation and that of an ordinary lottery is that here the number of "drawings" is uncertain, in that the number of fires that will occur during the period which the analysis is concerned with it is not known at the time of the decision and that in this "lottery" there are no prises, only losses. Despite these dissimilarities, thinking of an alternative in terms of a "lottery" is helpful, although the term "exposure" can be considered more appropriate in the present context. A particular fire exposure is defined here as an uncertain situation in which the number of fires that will occur in the building (or whatever) in question during a specific period of time is unknown and the consequences of any particular fire is uncertain.

Although there is considerable uncertainty regarding the outcome of a certain type of exposure in a particular building, i.e. the number of fires that will occur and their severity, it is possible to analyse the situation in such a way that exposures of different types (for example different building designs) can be compared. In evaluating different types of exposures, the concept of CE is helpful. As explained above, CE is the monetary value a particular uncertain situation is seen to possess, which means that for a particular type of exposure for which CE is calculated the decision maker should be willing, in terms of Bayesian decision theory, to pay any amount that is less than CE in order to avoid that type of exposure.

The crucial question here is how much money the decision maker would be willing to pay in order to *change* his/her exposure from that which the current building design involves to the type of exposure that would result from the decision maker having invested in additional fire safety measures. This monetary amount can be assessed by calculating CE for each of the two types of exposure and determining the difference between them. This value then is the *intrinsic* monetary value of the risk reduction that the fire safety investment involves.

## 3. Time preference

The discussion of decision theory above has dealt with *risk* preferences, such as in connection with CEs and with preferences for different outcomes, as represented by the utility values of the possible consequences. There is one additional preference that is of importance in the present context, that is for time preference.

A time preference can involve, for example, receiving a given sum of money today being regarded as better than receiving the same amount a year later. This is a matter dealt with by the methods for investment appraisal that are commonly employed such as the Present worth-method, the Annual worth-method, and the Future worth-method (see Canada and White [9]). Time preferences are considered important in the present context, a method similar to the discounting technique as used in the Present worth-method, for example, being suggested for representing the decision maker's preferences regarding the time at which a consequence occurs.

In order to calculate the CE for a certain type of exposure during a particular time period of interest, one needs first to calculate the expected utility associated with this exposure (see Eq. (1)). This requires that certain assumptions be made about the decision maker's preferences regarding the occurrence of more than one fire during a given period of time. In particular, it is assumed that the expected utility of $k$ fires during a given period of time, each of them with the expected utility $E(u)$, is $kE(u)$. This implies the utility of any given fire not being affected by the many other fires that occurred during the period.

The assumption just referred to enables one to calculate the expected utility of a particular type of fire exposure during a given time period $j$, likewise being assumed that the occurrence of the fires can be described by a Poisson process (see Section 6). Eq. (2) is used to calculate the expected utility ($E(u_j)$) for the type of fire exposure involved, $\lambda$ being the frequency of fire (in fires per year), $t_j$ the length of the time period considered (in years), $P(k)$ the probability of $k$ fire occurs during this period, and $E(u)$ the expected utility of any given fire.

$$E(u_j) = \sum_k E(u) \cdot k \cdot P(k) = E(u) \cdot \sum_k k \cdot P(k) = E(u) \cdot \lambda t_j \tag{2}$$

As discussed above, discounting methods used in traditional investment appraisal (see [9], for example) are employed to take account of the time preferences. Such methods involve the loss of a particular monetary amount 5 years from now, for example, being seen as less severe than the loss of the same amount at present. The intrinsic monetary value ($x$) of a loss that occurs $n$ years from now is assumed to be equal to a loss of $x/(1+i)^n$ today, $i$ being the interest rate that corresponds to the decision maker's time preferences. Dividing the period of time which is planned for into shorter time periods enables one to discount to the present level the intrinsic monetary value of the consequences that occur during each of these time periods. Usually, time periods of 1 year each are employed. This means that the utility of a fire that causes losses having the intrinsic monetary value of $x$ during the $j$th year of the period planned for it is calculated by discounting $x$ to the present and then calculating the utility of the discounted amount. Eq. (3) is used to calculate the utility $(u(x_{s,j}))$ of the loss $(x_{s,j})$, in the case of fire scenario $s$, occurs during the $j$th year of the time period of interest, $i$ being the interest rate used to discount the monetary values in question.

$$u(x_{s,j}) = u\left(\frac{x_{s,j}}{(1+i)^j}\right) \tag{3}$$

The expected utility of a given type of exposure can be calculated for a particular period of time by use of Eq. (4), in which $E(u_E)$ is the expected utility of a particular type of exposure, $n$ the number of years considered and $m$ the number of fire scenarios taken account of the building in question. The fire frequency here ($\lambda$) represents the expected number of fires per year, the time period ($t_j$) likewise being expressed in terms of years. When each time period is a year, the term $t_j$ in Eq. (4) can be disregarded.

$$E(u_E) = \sum_j \left(\lambda t_j \cdot \sum_s u(x_{s,j}) \cdot P(x_{s,j})\right), \qquad j = 1, \ldots, n, \quad s = 1, \ldots, m \tag{4}$$

The equation indicates that the expected utility of the type of exposure that is considered is calculated by summarising the expected utility over the years in question to yield the expected utility for the period as a whole that is planned for. The expected utility can readily be translated then into the CE, enabling the monetary value for the type of exposure in question to be calculated.

## 4. Uncertain estimates

Calculating the expected utility for a given type of exposure is not easy, however. The considerable uncertainty associated with the occurrence and spread of fire is a major reason for this. Various methods can be employed to deal with this uncertainty, QRA being a fruitful point of departure. In QRA based on the definition of risk proposed by Kaplan [10], one aims at specifying the accident scenarios that are representative for the building in question and at assessing their respective consequences and probabilities. In doing this, it is common to combine the probabilities of various events in an event tree, such as that “the sprinkler system succeeds in extinguishing the fire”, in such a way that the probability of a given fire scenario can be obtained.

Although uncertainty can be represented in ways other than by probabilities, such as by fuzzy measures [11], for example, use of probability measures seems to be the most fruitful approach in the present context [1]. In using probabilities to represent uncertainty, it is important to take account of the interpretation of probability that one explicitly or implicitly adopts. It has been argued that the subjective interpretation of probability is particularly useful in risk analysis [12]. Such an interpretation is the one adopted in the present paper. A subjective interpretation means a probability being regarded as a degree of belief in some proposition or event. Use of this interpretation provides considerable flexibility when a risk analysis is performed, and can also be considered as essential for the practical application of the methods suggested here.

In performing a QRA of a factory of some sort, one is very likely to feel uncertain about the estimates of various parameters, such as probabilities and frequencies. From a Bayesian standpoint, ambiguity regarding a probability or frequency estimate should be represented by a probability distribution defined over all possible values of the parameter in question (for example, see [13]). An example of such a distribution will be given shortly. In Bayesian decision theory, however, ambiguity of this sort is assumed to not affect which decision alternative is best, or how much the decision maker should be willing to pay (the CE) in order to avoid a particular type of exposure. According to that theory, the expected value of the distributions are the only values needed to determine the CE of a given type of exposure. The author has argued [1], however, that in a context such a the present a Bayesian evaluation based on expected utilities is in need of being complemented by a further evaluation, one aimed at determining whether the choice of which decision alternative is best is robust. In brief, the concept of robustness implies that if a plausible degree of change in the assessment of the consequences and the probabilities is made, the alternative regarded as best will not change. The key to determining whether a decision alternative is robust is to relate the uncertainty of the probabilities and of the utilities of the consequences to the difference in expected utility between the decision alternative in question and the other alternatives.

To exemplify the approach suggested, consider a choice between three fire protection alternatives for which the uncertainty regarding the values of the probabilities and of the utilities of the consequences as assessed in the model can be expressed as distributions that represent the decision maker’s belief regarding their values. Assume in addition that the result, when the expected utility of the different alternatives is calculated is that alternative 1 has the highest expected utility, followed by alternatives 2 and 3 in that order.

According to Bayesian decision theory, alternative 1 is thus the alternative that the decision maker should choose. Assume, however, that the difference between alternatives 1 and 2 is only slight. If one expresses the difference between the expected utility of the two alternatives as a probability distribution, it could look likes as in Fig. 1. One can see that most of the mass of the probability distribution denoting the difference in probability $(E(U_1) - E(U_2))$ is located on the positive part of the horizontal axis, indicating alternative 1 to have the highest expected utility. However, there is also a significant part of

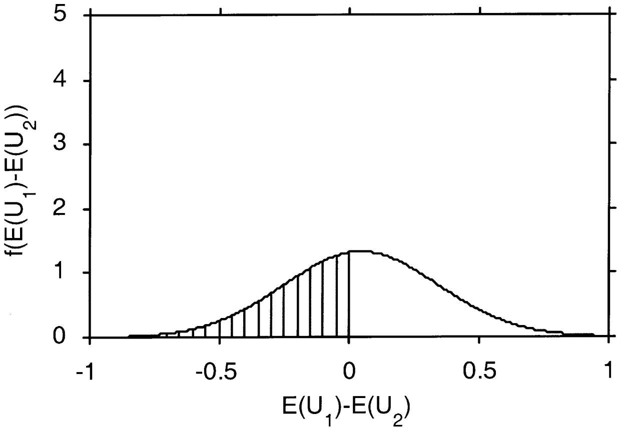

Fig. 1. Probability distribution representing the difference in expected utility between alternatives 1 and 2.

the probability distribution located within the negative region, indicating alternative 2 to have the highest utility. This would imply, loosely speaking, that a reasonable change in the assessments of the probabilities and of the consequences could result in the alternative with the highest expected utility changing. This is a situation in which the alternative regarded as the best (alternative 1) is not deemed to be robust. If, on the other hand, the decision maker only had alternatives 1 and 3 to choose between, the choice of alternative 1 would likely have been considered robust, since if one looks at the distribution showing the difference in expected utility between alternatives 1 and 3 (the distribution illustrated in Fig. 2) one can see that the entire mass of the distribution is located in the positive region along the horizontal axis.

In practical applications of the method just discussed, a decision is generally deemed to be robust if 95% of the distribution representing the difference in utility between two

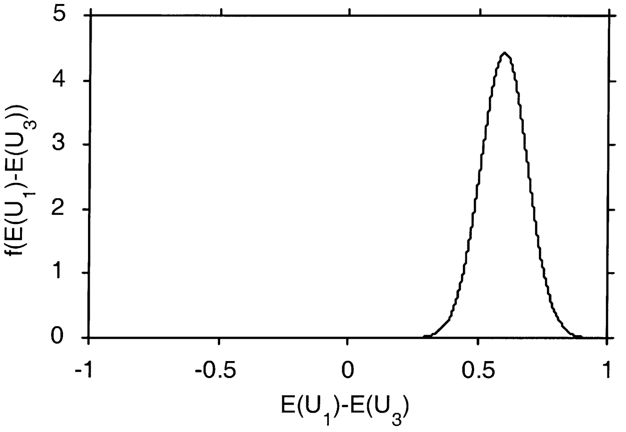

Fig. 2. Probability distribution representing the difference in expected utility between alternatives 1 and 3.

alternatives indicates one and the same alternative to be best. This approach is only one that is recommended, its being up to the individual decision maker to choose a value that he/she feels comfortable with.

## 5. Fire scenarios and fire frequency

Through QRA, one can estimate the frequency of fire in a particular building and arrive at a plausible set of possible fire scenarios together with their respective conditional probabilities of occurrence (conditional on the event that a fire has occurred). The technique for doing this can vary considerably, the method described here being one found to be useful in two real-world analyses the author has carried out.

The basic idea of the method to be described is to divide up the building in question into suitable areas, preferably coinciding with the various fire compartments of the building. For each such area, a model of how a fire might develop needs to then be created. In the two real-world analyses referred to, an event tree technique was used to indicate the different fire scenarios that were considered suitable in the buildings in question and to calculate the conditional probability of each scenario, given that a fire had occurred in that area. In the event tree, different events that could mitigate or affect in some other way the spread of a fire were included. The events can be considered roughly to be of five different kinds, those pertaining to fire potential, to employees, to active systems (such as sprinklers), to passive systems (such as fire compartments) and to the actions of the fire department. Fire potential concerns such matters as the fact that if a fire occurs in an area where the amount of combustible material is limited it might consume all the material there and being extinguished before causing any significant damage. All the relevant events that can mitigate or in any other way affect the development of a fire must be included in the event tree. Examples of such trees are given in [8].

The next step, after the model have been created, is to estimate the probabilities of the different events. As has already been indicated, these probabilities can sometimes be very difficult to estimate, particularly when there is only limited information about the events and the events are concerned with phenomena that are difficult to create models for. An example of such an event is “those employed in the building succeed in extinguishing the fire”. Since the decision maker is likely to feel very uncertain in estimating probabilities of this sort it is advantageous to employ a decision analytic framework that allows probabilities to be expressed in an imprecise way. As shown in Section 6, Bayesian methods can be used to reduce the uncertainty regarding the frequencies and probabilities considerably.

When the model for the development of a fire in the building is complete, one can create a list of all relevant fire scenarios, their consequences and conditional probabilities. Besides having the list described above, one needs to also have a model of how often a fire can be expected to occur, or of the frequency of fire. A good point of departure in estimating the frequency of fire in a building is to consider the results of investigations of the frequency of fire in buildings of different categories, as presented by Fontana et al. [14]. In this reference, estimates of the frequency of fire per square meter in buildings of various types are given. This information can help one arrive at an estimate for a specific building. Fire statistics from the building in question can also contribute to this. Bayesian methods for the incorporation of new evidence into estimates through the use of Bayes' theorem are useful here. This will be discussed and exemplified in the next section.

## 6. Bayesian updating

As already indicated, one easily feels uncertain about the value of a probability of an event that affect the outcome of a fire. Accordingly, instead of assigning a precise value to the probability in question, it may be better to employ a probability distribution to represent one's belief regarding the value the probability has. One benefit of doing this is, besides it enables the decision maker to express his/her uncertainty in a more adequate way, it enables the information from different sources to be combined in estimates made by the use of Bayesian methods. In the present context, such information can be information regarding a limited number of fires that have occurred in the building of interest. Whereas this information alone is usually not sufficient to serve as a basis for estimating the different probabilities in the model, it becomes much more useful if combined with other sources of information, such as expert judgement and the like. How different types of information can be combined in this way in situations of different kinds that are likely to arise in a context such as the present one of decision analysis with respect to fire safety has been discussed by Johansson [15]. Here, only one of these possible situations will be discussed, that of estimating the frequency of fire in a particular building.

The basic principles of Bayesian methods employed when incorporating information from different sources into a probability assessment have been dealt with detail in [13]. Stated briefly, one begins with a *prior* probability distribution, one that represents the decision maker's belief regarding the uncertain parameter *before* any of the evidence has been taken into account. This prior distribution is updated then using the information in question, which could include information regarding a particular fire in the building. This updated distribution, termed the *posterior* probability distribution, is obtained by use of Bayes' theorem.

In a case study carried out concerning a cold-rolling mill belonging to the company Avesta Sheffield, a study in which the methods discussed in this paper were employed, one of the uncertain parameters was the frequency of fire in the building. The only information available regarding this parameter was the number of fires that had occurred during the past 6 years. In order to obtain an estimate of the frequency of fire in the building, a so-called diffuse prior distribution was employed. This is a distribution that represents there being no strong belief in any particular value of the parameter in question. Although in the case study both a discrete and a continuous prior distribution were used to represent the frequency of fire, only results involving use of the continuous distribution will be presented here. This distribution was chosen from the class of Gamma distributions since such distributions are flexible and are the conjugate family of distributions when the parameter of interest is the expected number of occurrences of some uncertain event, such as a fire, and when the number of events per time interval can be described by a Poisson distribution.

If fires can be assumed to occur independently of each other and to occur with a constant intensity, the Poisson distribution can be used to calculate the probability that some given number of fires will occur during a specified period of time. Since both these assumptions

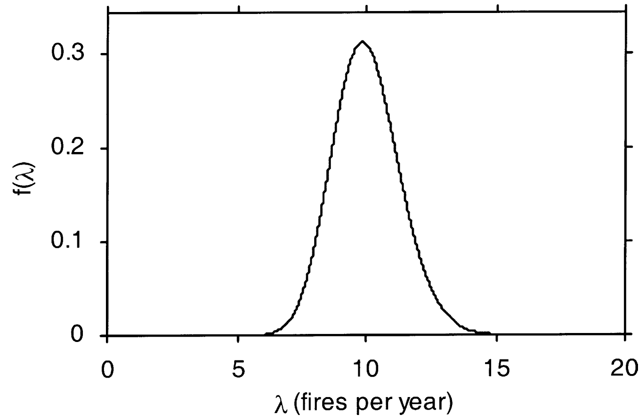

Fig. 3. The posterior distribution of the frequency of fire (λ) in the cold-rolling mill.

appeared reasonable the Poisson distribution was used to calculate the probability that a given number of fires would occur in the cold-rolling mill, given a particular frequency of fire in the building. The diffuse prior distribution employed was a Gamma distribution in which both parameters, $s$ and $m$, were equal to 0. According to Lindley [13], this is the Gamma distribution to be used for representing vague prior knowledge.

The information contained in the fire statistics from the cold-rolling mill indicated that during a period of 6 years there had been 60 fires, which might be seen as many in a building of the type and size of the present one (see Section 8 for a description). However, most of the fires were very small and were extinguished quickly. Many of them occurred in the machines, where they could be extinguished by automatic suppression systems. Nevertheless, all such incidents were counted in estimating the frequency of fire in the building. Using this information to update the diffuse prior distribution resulted in a posterior distribution that looked like the one shown in Fig. 3. It is a Gamma distribution with the parameters $s = 60$, and $m = 6$. This distribution represents the decision maker's belief regarding the frequency of fire in the mill after the information contained in the fire statistics had been taken into account. Looking at the figure showing the posterior distribution, one can draw the rough conclusion that the frequency of fire in the mill is likely to be somewhere between 6 and 14 fires per year.

One can use the posterior distribution shown in Fig. 3 as a prior distribution in a later updating procedure if additional evidence becomes available. Note that the updating procedure just exemplified can be used for all uncertain probabilities contained in the model of the different fire scenarios in any given building. The only difference as compared with the example just described is that instead of a Poisson distribution some other distribution might be needed, depending upon the information one makes use of. The author has discussed and exemplified some of the most common situations likely to be encountered in a context such as the present one in [15]. Note that Bayesian methods can also be employed in connection with expert judgements to incorporate them in a formal way into assessments of probability here. Thus, even without any statistical evidence regarding fire in the building of interest, uncertainty regarding the parameters of concern can be reduced by use of expert judgement.

## 7. Investment appraisal

The method used here for the investment appraisal of a fire safety investment is based on various methods discussed above. It is similar to the Present worth-method in that cash flows are discounted to the present, i.e. to the time when the decision is made, so that cash flows occurring at different times are comparable.

That which is aimed at is an estimate of the Risk-adjusted net present value of the investment, which in turn involves taking account of the risk reduction that the investment provides, as well as costs of a more fixed or certain character, such as those for maintenance. The Risk-adjusted net present value being defined here as the monetary value equivalent to making the investment in question.

Obtaining this estimate requires that the CE of installing the fire protection system be estimated. As indicated above, this involves first choosing the period of time to which use of the fire protection measure is to apply and then performing a risk analysis in which the probability of each of the possible fire scenarios, as well as the consequences of each, are calculated, both for the building in its present state and when equipped with the fire safety system.

Performing a decision analysis using a risk analysis as a point of departure takes account of the decision maker's preferences with respect not only to the possible consequences of a fire, but also to risk in general, as well as to the occurrence of the consequences in question at differing times during the period that is planned for.

From this, finally, one can obtain the Risk-adjusted net present value of the investment, or the monetary value the investment has when both the intrinsic value of the risk reduction and more certain or fixed costs such as those of maintenance are taken into account.

## 8. Case study: Avesta Sheffield

In 1999 an analysis using the methods described above was performed for a cold-rolling mill belonging to the company Avesta Sheffield. The mill had a production capacity of approximately 100,000 t of stainless steel per year. The factory was approximately 15,000 $m^2$ in size. The analysis that was performed concerned investment in a water sprinkler system for the entire building.

In carrying out the analysis, two event trees were created, each representing the fire scenarios that had been identified in the building. The two event trees represented the building with and without the sprinkler system, respectively. Using these event trees in combination with estimates of the probabilities of the different events in the event trees and assessments of the consequences resulting from the different fire scenarios enabled the intrinsic monetary value of the risk reduction that an investment in a sprinkler system would involve to be calculated.

Since an investigation of the company's risk tolerance was outside the scope of the study, assumptions had to be made regarding it. As it turned out, however, using a risk neutral utility function or an exponential utility function (signifying risk aversion), different values for risk tolerance being inserted into it, did not change the alternative found to be best. The results reported here were obtained using the risk-neutral utility function.

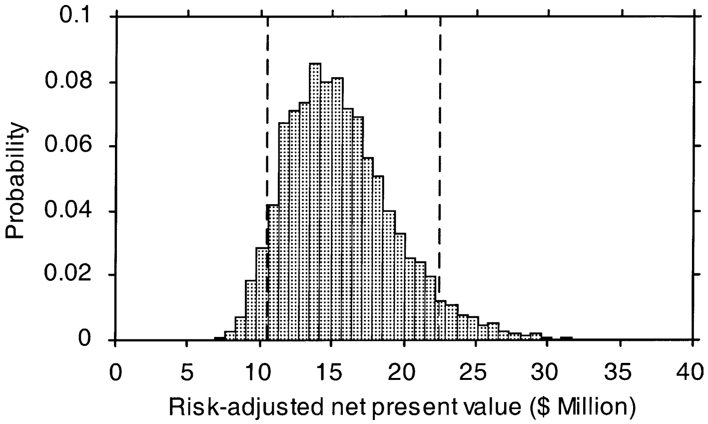

Fig. 4. The probability distribution representing the Risk-adjusted net present value for the investment in a sprinkler system in the cold-rolling mill.

The probabilities and consequences employed in the model for fire spread were uncertain and were thus expressed as probability distributions (see Sections 4 and 6), which meant that the Risk-adjusted net present value was also uncertain. Monte Carlo-simulation involving 5000 iterations was employed for estimating the distribution describing the uncertainty regarding the Risk-adjusted net present value. The results of these iterations are shown in the histogram in Fig. 4. The two dashed lines in the figure represent the boundaries between which 90% of the values obtained by the Monte Carlo-simulation are located. The Risk-adjusted net present values at these boundaries are $ 10.2 million and $ 22.5 million, respectively. The monetary sums given in this section, originally in Swedish crowns (SEK), were converted to US dollars at the rate of $ 1 to 10 SEK. In calculating the Risk-adjusted net present value, the costs taken into account were the initial investment costs, estimated to be $ 250.000, and the annual maintenance costs, estimated to be $ 5.000 per year. In the primary analysis, it was assume that a period of 40 years was planned for in connection with the sprinkler system and a discount rate of 20% was employed.

Since the basis for the evaluation is the Bayesian decision theory, the value that should be used to evaluate the investment is the expected value of the resulting distribution, which in this case means that a good approximation for this value would be the mean value of the result from the 5000 Monte Carlo-simulations. This value is $ 15.6 million, which indicates that the investment is very “profitable” and should be made. No considerations of price changes were taken in the calculations.

To determine whether the decision was robust, it was decided that would be the case if 95% of the Risk-adjusted net present values from the Monte Carlo-simulation indicated that the investment should be made. Looking at the histogram in Fig. 4, it is clear that the decision is robust since all of the values are on the positive region of the horizontal scale.

Although the planning period and the discount rate were provided by the decision maker (Avesta Sheffield), it was considered useful to perform a sensitivity analysis of these parameters. Fig. 5 presents the results of the sensitivity analysis for the period in question.

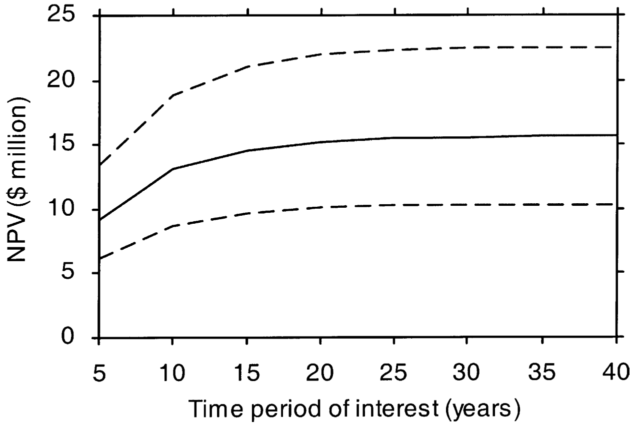

Fig. 5. The effect on the Risk-adjusted net present value of changing the time period of interest.

The two dashed lines represent the boundaries between which approximately 90% of the resulting distribution of the Risk-adjusted net present value lies. The figure shows the Risk-adjusted net present value to be nearly the same for all periods longer than approximately 20 years. In the case of planning for shorter periods of time, the value is less but is still positive and robust (the lower dashed line is in the positive region along the vertical axis), implying that use of a shorter time period of interest would not affect the attractiveness of the investment. The same type of sensitivity analysis was performed for the discount rate. It showed that if a lower discount rate than the one used in the primary analysis (20%) was employed, the Risk-adjusted net present value would be greater than in the primary analysis. If the discount rate was set at a level higher than 20%, the Risk-adjusted net present value was also positive for all the values employed (i.e., up to 50%). It was thus concluded that the result was stable with respect to the discount rate as well.

Note that the high Risk-adjusted net present value of the sprinkler system does not mean that the investment's "pay-back time" is short. It is possible, though unlikely, that the sprinkler system will never need to be used during the 40-year period that is planned for. In such a case, of course, the investment would be a bad one because of never having been needed. Since when one makes the decision, however, it is impossible to know whether the system will be needed or not, one has to rely on estimates of fire frequency and on the modelling of fire spread. The model described in this paper gives the result that the sprinkler can be expected to be very useful as a risk reducing measure in the Avesta Sheffield building. Since a large fire can cause considerable losses (in the order of hundreds of million dollar), lowering the probability of such a consequence slightly has a large effect on the result. This is why the Risk-adjusted net present value is so high.

In comparing the investment appraisal described here with a similar one performed in a factory belonging to the company Asea Brown Boveri (ABB) one can note that investment in the sprinkler system located in the present case in a cold-rolling mill is more "profitable" than the investment was in the case of the ABB factory [8]. For the ABB building the

Risk-adjusted net present value was calculated (by use of the same method) to be $ 3.1 million. This difference in Risk-adjusted net present value can be explained by the fact that the passive fire protection in the cold-rolling mill (fire-rated walls, etc.) is not as good as that in the ABB building. This means that if a fire grow large in a fire compartment it is more likely to spread to other compartments in the cold-rolling mill than in the ABB building. In addition, the losses associated with fires are smaller in the ABB building than in the cold-rolling mill, and in the ABB building there are other kinds of fire protection systems (automatic smoke detection in the entire building, for example) which the cold-rolling mill does not possess. Because of these differences, the relative increase in safety which investing in a sprinkler system would provide is greater for the Avesta Sheffield than for the ABB building.

## 9. Conclusions

Use of QRA for the appraisal of fire safety investments, using methods based on Bayesian decision theory, has been discussed. Particular attention has been directed at the problem of evaluating losses due to fires that occur at different times, use of a method similar to that of the discounting of cash flows being suggested for modelling the decision maker's time preferences. Taking account of the decision maker's time preferences, risk preferences, and preferences regarding various monetary consequences of fire, as shown by the corresponding utility functions, enables the Risk-adjusted net present value of an investment in fire safety, or the assessed monetary value of having made the investment, to be calculated. If and only if this latter value is positive, should the investment be made.

Calculating the Risk-adjusted net present value of an investment in fire safety is based on QRA. Since many of the probabilities used here are uncertain, the Risk-adjusted net present value obtained is uncertain. According to Bayesian decision theory, uncertainty of this sort should not affect the decision made. In the present context it has been judged to be beneficial, however, to relate the uncertainty regarding both the probabilities and the consequences to the Risk-adjusted net present value, the latter being represented as a probability distribution, so as to indicate how certain the decision maker is regarding the Risk-adjusted net present value of the investment.

A method of reducing the uncertainty regarding the occurrence and spread of fire is dealt with in the paper. The method employs Bayesian methods for integrating specific information concerning the building of interest with other types of information, such as expert judgement, general statistics, and the like.

A real-world problem dealing with investment in a water sprinkler system for a cold-rolling mill was also analysed, showing in practical terms how the approach described can be applied and how the results can be presented in a meaningful way to the decision maker.

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